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DIFFERENTIAL EQUATIONS

BMTC-132 - 2nd Semester

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Central Universities, State Universities, IGNOU and Other Open Universities

By:
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MRP ₹ 200/-

Published by:

NEERAJ PUBLICATIONS

Sales Office : **1507, 1st Floor, Nai Sarak, Delhi-110 006**
E-mail: **info@neerajignoubooks.com**
Website: **www.neerajignoubooks.com**

Typesetting by: ***Competent Computers*** | *Printed at*: ***Novelty Printer***

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CONTENTS

DIFFERENTIAL EQUATIONS

Sample Question Paper–1 (Solved) 1-2

Sample Question Paper–2 (Solved) 1-2

Sample Question Paper–3 (Solved) 1-2

Sample Question Paper–4 (Solved) 1-2

Sample Question Paper–5 (Solved) 1-2

S.No.	Chapter	Page
1.	The Functions of Variables	1
2.	Limits and Continuity of Functions	7
3.	Partial Derivatives of First Order	16
4.	Partial Derivatives of Higher Order	26
5.	Chain Rule and Homogenous Function	35
6.	The Nature of Differential Equations	49
7.	Methods of Solving First Order and First Degree	57
8.	Linear Type Differential Equations	73
9.	Differential Equations of First Order But Degree Greater than One	80

S.No.	Chapter	Page

10. Linear Differential Equations of Higher Order 91

11. Method of Undetermined Coefficient in
Higher Order Differential Equations 98

12. Method of Variable Coefficient 108

13. Method of Operators 117

14. Simultaneous Type of Differential Equations 132

15. Total Differential Equation 142

16. Homogenous and Non-Homogenous
Linear Partial Differential Equation 151

17. Partial Differential Equations of First Order 161

Sample Preview of the Solved Sample Question Papers

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Sample Question Paper 1 (Solved)

DIFFERENTIAL EQUATIONS

Time: 3 Hours] *[Maximum Marks: 100*

***Note:** Attempt the following questions.*

Q. 1. ***(a)*** **If $a = (1, 2)$ and $b = (2, 0)$ are two point in R^2 then find $|x - y|$ and $|3x - y|$ where $x = a - 2b$ and $y = 2a + b$.**

Ans. Ref.: See Chapter-1, Page No. 4, Q. No. 7.

(b) $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{x^2 - y^2}{x^2 + y^2} = 0.$

Ans. Ref.: See Chapter-2, Page No. 10, Q. No. 6.

Q. 2. ***(a)*** **Find f_{xx}, f_{yy} given that**

$f(x, y) = \sin xy$

Ans. Ref.: See Chapter-4, Page No. 27, Q. No. 2.

(b) **Verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ for each of the following functions.**

Ans. Ref.: See Chapter-4, Page No. 31, Q. No. 12.

Q. 3. ***(a)*** **Solve : $(x^4 + y^4)dx - xy^3 dy = 0$**

Ans. Ref.: See Chapter-7, Page No. 69, Q. No. 10.

(b) $\frac{dy}{dx} - y = 6\cos 2x$

Ans. Ref.: See Chapter-8, Page No. 77, Q. No. 7 *(a)*.

Q. 4. ***(a)*** **Verify if the function $y = \frac{1}{4}\sin 4x$ is a unique solution of the initial value problem.**

$y'' + 16y = 0$

$y(0) = 0, y'(0) = 1.$

Ans. Ref.: See Chapter-10, Page No. 93, Q. No. 1.

(b) **Solve the following equation:**

$$y' = y + \frac{e^x}{x}, x \in [1, \infty[$$

Ans. Ref.: See Chapter-8, Page No. 76, Q. No. 4 *(a)*.

Q. 5. Determine the general solution of the following equations:

(a) $\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 4x^2$

Ans. Ref.: See Chapter-11, Page No. 100, Q. No. 2*(a)*.

(b) $\frac{d^3 y}{dx^3} + 4\frac{dy}{dx} = x$.

Ans. Ref.: See Chapter-11, Page No. 100, Q. No. 2*(b)*.

Q. 6. ***(a)*** **Show that the orthogonal trajectories on the conicoid $(x + y)z = 1$ of the conics in which it is cut by the system of planes $x - y + z = k$ (k is a parameter) are the curves of intersection with the surface $x + c_1 = z + \frac{1}{2z} - \frac{1}{6z^3}$.**

Ans. Ref.: See Chapter-14, Page No. 140, Q. No. 7.

(b) **An electric circuit consists of an inductance of 0.1 henry, a resistence of 20 ohms and a condenser of capacitance 25 microfarads. What will be the form of the governing equation if there is a variable electromagnetic force of 100 cos 200t volts?**

Ans. Ref.: See Chapter-14, Page No. 141, Q. No. 9.

Q. 7. ***(a)*** **A function $f : R^2 \to R, f(x, y) = x^2 + xy + y^2$ then find out f_x and f_y.**

Ans. Ref.: See Chapter-3, Page No. 18, Q. No. 1.

(b) Let $f : R^2 \to R$ be defined by

$$f(x, y) = \begin{cases} 1 & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$$

Show that $f_x(0, 0)$ as well as $f_y(0, 0)$ does not exist.

Ans. Ref.: See Chapter-3, Page No. 19, Q. No. 9.

Q. 8. Solve the following equations:

(a) $(y')^2 - 4 = 0$

Ans. Ref.: See Chapter-9, Page No. 85, Q. No. 4 *(a)*.

(b) $x = y + a \ln p$

Ans. Ref.: See Chapter-9, Page No. 85, Q. No. 3 *(b)*.

Q. 9. Solve the following equations:

(a) $[(x+a)^2 D^2 - 4(x+a)D + 6]y = x$

Ans. Ref.: See Chapter-12, Page No. 113, Q. No. 9 *(a)*.

(b) $[(1+x)^2 D^2 + (1+x)D + 1)y = 4\cos[\ln(x+1)]$

Ans. Ref.: See Chapter-12, Page No. 113, Q. No. 9 *(b)*.

Q. 10. (a) Find the general integral of $(D^2 + DD' - 2D'^2)z = e^{x+y}$.

Ans. Ref.: See Chapter-16, Page No. 157, Q. No. 16.

(b) Solve the following partial differential equations:

$$(3DD' - 2D'^2 - D')z = \sin(2x + 3y)$$

Ans. Ref.: See Chapter-16, Page No. 158, Q. No. 20.

■■

Sample Preview
of
The Chapter

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DIFFERENTIAL EQUATIONS

The Functions of Variables

1

INTRODUCTION

A real valued function f of x, y, z is a rule for manufacturing a new number written $f(x, y, z)$ from the values of a sequence of independent variables x, y, z.

The function f is called a real valued function of two variables if there are two independent variables, a real valued function of three variables if there are three independent variables and so on.

A vector valued function also required to as a vector function, is a mathematical function of one or more variables whose range is a set of multi dimensional vectors. The input of a vector valued function could be a scalar or a vector.

In this chapter, we define R^n and describe its algebraic structure. We also introduce the notion of a distance between two points of R^n and deduce its elementary properties. We define a function of several variables and by giving various examples of such functions.

CHAPTER AT A GLANCE

THE SPACE R^n

Real valued functions of several independent real variables are defined much the same way one would imagine from the single variable case. The domains are sets of ordered in tuples or real numbers. Let

$R^n = \{(x_1, x_2, ... x_n)\} \, |x_i \in R, i = 1, 2 ... n\}$

Cartesian Product: The Cartesian product of two set X and Y is defined to be the set of all points (x, y) where $x \in X$ and $y \in Y$. It is denoted by $X \times Y$.

Thus $X \times Y = \{(x, y) \, |x \in X, y \in Y\}$

For Example: If $X = (a, b, c)$ and $Y = (c, d)$ then

$X \times Y = \{(a, c) \, (a, d) \, (b, c) \, (b, d) \, (c, c) \, (c, d)\}$

If $X = R$, $Y = R$ then $X \times Y = R \times R$

$= R^2 \, \{(a, b) \, |a \in R, b \in R\}$

If $X_i = R$ for all i, $1 \le i \le n$ then

$X_1 \times X_2 \times ... X_n = R \times R \times ... \times R$ (n times)

$= R^n$

$= \{(a_1, a_2 ... a_n) \mid a_i \in R, 1 \le i \le n\}$

is called the Cartesian product of n-copies or R.

If $A = (a_1, a_2 ... a_n)$ is any point of R^n then a_i is called ith co-ordinate or i-th component of A.

Algebraic Structure of R^n

A zero vector denoted 0, is a vector of length 0 and thus, has all component equal to zero. It is the additive identity of the additive group of vectors.

Sum of two vectors: Consider two vectors in R^n

$x = \{x_1, x_2 ... x_n\}$

$y = \{y_1, y_2 ... y_n\}$

We define the sum vector $x + y$ as

$x + y = \{x_1 + y_1, x_2 + y_2, ... x_n + y_n\}$

For example, consider R^3 the two vectors

$x = \{7, 8, 9\}$ and

$y = \{2, 4, 7\}$

We have

$x + y = \{7 + 2, 8 + 4, 9 + 7\}$

$= \{9, 12, 16\}$

so $x + y \in R^n$, that is through the sum a new element of R^n has been structured.

We have therefore, introduced in R^n two operations sum and scalar multiplication.

Let x, y and z be three vectors in R^n. We have

(i) $x + y \in R^n$ (R^n is closed with respect to the sum)

(ii) $x + y = y + x$ (Commutative Property)

(iii) $x + 0 = x$ (existence of a neutral element for the sum)

(iv) $x + (-x) = 0$ (existence of the opposite of each vector)

Now the scalar multiplication: Let $x, y \in R^n$ and $\alpha, \beta \in R$. We have

(i) $ax \in R^n$ (R^n is closed with respect to scalar multiplication)

(ii) $\alpha(x + y) = \alpha x + \alpha y$ (Distributive Property)

(iii) $(\alpha + \beta) x = \alpha x + \beta x$

(iv) $\alpha(\beta x) = (\alpha\beta) x$ (Associative Property)

Distance in R^n: Let $A = \{a_1, a_2, ... a_n\}$ and $B = \{b_1, b_2 ... b_n\}$ be two points R^n, then distance

$|A - B| = \sqrt{\sum_{i=1}^{n}(a_i - b_i)^2}$ if $n = 1$ then $|A - B| = \sqrt{(a-b)^2}$. Show the distance between the points a and b on the real line. If $n = 2$ then $|A - B| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ Show the distance between two points (a_1, a_2) and (b_1, b_2) in Cartesian plane. If we study in 3-dimensions would recognise for n = 3 is

$$|A - B| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

Show the distance between two points $(a_1 a_2\ a_3)$ and $(b_1\ b_2\ b_3)$ in space.

Theorem 1 : (Cauchy's Inequality): If $a_1, a_2 ... a_n$, $b_1, b_2, ... b_n$ are any $2n$ real numbers then

$$\left|\sum_{i=1}^{n} a_i b_i\right| \leq \sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}$$

Proof: To prove this form of the inequality, consider the following quadratic polynomial in z

$(a_1 z + b_1)^2 + ... + (a_n z + b_n)^2$

$= [\Sigma (a_1)^2 . z^2 + 2 (\Sigma (a_i . b_i) . z + \Sigma (b_i)^2]$

Since it is non-negative it has at most one real root in z that is

$(\Sigma (a_i b_i))^2 - \Sigma a_i^2 . \Sigma b_i^2) \leq 0$

Which yields the Cauchy Inequality.

An equivalent proof for R^n starts with the summation below:

$$\sum_{i=1}^{n}\sum_{i=1}^{n}(a_i b_j - a_j b_i)^2 = \sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2 - \left(\sum_{i=1}^{n} a_i b_i\right)^2$$

Because the left hand side of equation is a sum of the squares of real number it is greater than or equal to zero thus,

$$\left|\sum_{i=1}^{n} a_i b_i\right| \leq \sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}$$

and the proof is complete.

Theorem 2 : (Triangle Inequality) For any three points x, y, z in R^n

$|x - y| \leq |x - z| + |z - y|$

Proof: We simply compute as follows:

$|x + y|^2 = (x + y)\ (x + y)$

$= |x|^2 + 2x \,.\, y + |y|^2$

$\leq |x|^2 + 2|x\ y|\ |y|^2$

$\leq |x|^2 + 2|x||y| + |y|^2$

Using Schwarz Inequality

$|x + y| \leq |x| + |y|$

If using three points x, y, z in R^n then

$|x - y| = |(x - z) + (z - y)|$

$|x - y| = |x - z| + |z - y|$

An n-dimensional open ball of radius r is the collection of points of distance less than r from a fixed point in euclidean n-space. Explicitly the open ball with centre x and radius r is defined by

$B_r(x) = \{y : |y - x| < r\}$

The open ball for $n = 1$ is called an open interval and the term open disk is sometimes used for $n = z$.

FUNCTIONS FROM R^n TO R^m

A real valued function of n variables is a function that takes as input n real number, commonly represented by the variables $x_1, x_2 ... x_n$ for producing another real number, the value of function commonly denoted $f(x_1, x_2, x_3 ... x_n)$.

Let X be a non-empty subset of R^n where $n \geq 1$. To calculate D to R^m is called a vector valued function of n variable with domain D.

Let $f : D \to R^m$ and

$x = \{x_1, x_2, ... x_n\}$ where $x \in D$,

$f(x) = (x_1, x_2 ... x_n)$

Let x and y two variables which is belong to R, then function

$f(x, y) = \sin x + \cos y \qquad \therefore f(x, y) \in D$

If $D = [-1, 1] \times [-1, 1]$ the value of function

$f(x, y) = \sin^{-1} x \cos^{-1} y$

When D be open sphere radius 1 and centre (0, 0, 0) in R^3 then

$$f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$$

$f(x, y, z)$ is a real valued function of 3 variables (x, y, z) with domain D.

Let $f : R \to R^2$, in a vector based function of one variable then

$f(x) = (x, 0)$

If vector valued define 2 variables to 3 variables function for (x, y) in $R^2 \to R^3$

$g(x, y) = (x, y, 0)$

If consider n variables for polynomial expression then variables.

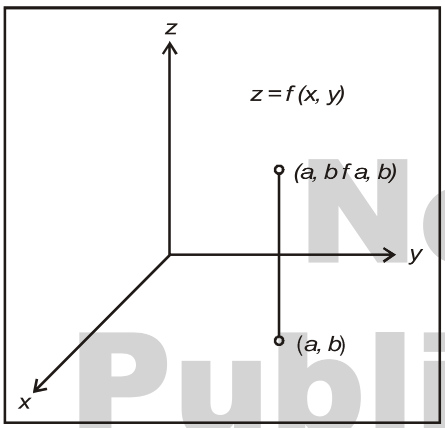
$$x = x_1, x_2, \ldots x_n$$

expression

$$\sum a\, k_{li}\, k_{zi} \ldots k_{ni}$$

where k_i are non-negative integers.

The graph of the function f of two variables is the set of points $(x, y, f(x, y))$ in three dimensional space where we restrict the values of (x, y) to lie in the domain of f. In other words, the graph is the set of all the points (x, y, z) with $z = f(x, y)$



Sum of Two Functions

Let $f : X_1 \to R^m$ and $g : X_2 \to R^m$ where X_1 and X_2 are subset of R^n. If $X = X_1 \cap X_2 \neq \phi$ then

$(f + g)(x) = f(x) + g(x)$

function f and g define sum of the two vector valued functions $(f + g)$

Product of Two Functions

Let $f : X_1 \to R^m$ and $g : X_2 \to R^m$ then

$$X = X_1 \cap X_2 \neq \phi$$

where X_1 and X_2 are subset of R^n.

$(fg)(x) = f(x)\, g(x)$

(fg) is the product of the two real valued functions f and g.

Quotient of Two Functions

Suppose f and g be real value functions then quotient of the functions f and g is

$$(f/g)(x) = \frac{f(x)}{g(x)}$$

where $D = \{x \mid x \in D, g(x) \neq 0\} \neq \phi$.

CHECK YOUR PROGRESS

Q. 1. Let $x = a_1 + a_2 - 2a_3$ and $y = 2a_1 - a_2 + a_3$ where a_1, a_2, a_3 are the unit vector. Find $|x + 2y|$, $|x + y|$.

Sol. $x + 2y = (a_1 + a_2 - 2a_3) + 2(2a_1 - a_2 + a_3)$
$= 5a_1 - a_2$
$= (5, -1, 0)$

$\therefore \quad |x + 2y| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$.

Similarly

$x + y = a_1 + a_2 - 2a_3 + 2a_1 - a_2 + a_3$
$= 3a_1 - a_3$
$= (3, 0, -1)$

$\therefore \quad |x + y| = \sqrt{3^2 + (-1)^2}$
$= \sqrt{10}$.

Q. 2. Show that the closed sphere with centre (2, 3, 7) and radius $10R^3$ is contained in open cube.
$P = \{(x, y, z\} : |x - 2| < 11, |y - 3| < 11, |z - 7| < 11\}$

Sol. We have

$S = \{(x, y, z\} \in R^3 : |(x - 2)(y - 3)(z - 7) < 10\}$

Now

$(x, y, z\} \in S = \sqrt{(x-2)^2 + (y-3)^2 + (z-7)^2} < 10$

$\Rightarrow (x - 2)^2 + (y - 3)^2 + (z - 7)^2 < 10$

$\Rightarrow (x - 2)^2 < 10, (y - 3)^2 < 10, (z - 7) < 10$

$\Rightarrow |x - 2| < 10, |y - 3| < 10, |z - 7| < 10$

$\Rightarrow S \subset P$

Q. 3. Verify the following for $x = 3$ and $y = 10$ $|2x - 3y - 5| \leq |2x - 3| + |3y + 2|$

Sol. We have given

$|2x - 3y - 5| \leq |2x - 3| + |3y + 2|$

For $x = 3$, $y = 10$ then

$|6 - 30 - 5| \leq |6 - 3| + |30 + 2|$

$\Rightarrow \quad |-29| \leq |3| + |32|$

$\Rightarrow \quad 29 \leq 3 + 32$

$\Rightarrow \quad 29 \leq 35$

Q. 4. Find *fog* and *gof*, if they exist for the function given by

$f(x, y, z) = (e^x, \ln(x^2 + y^2 + 1), z^2)$

$g(x, y, z) = (x + y, zy, 5z)$

Sol. We have

$gof(x, y, z) = g[f(x, y, z)]$
$= g[e^x, \ln(x^2 + y^2 + 1), z^2)$
$= (e^x + \ln(x^2 + y^2 + 1), 2\ln(x^2 + y^2 + 1), 5z^2)$

and $fog(x, y, z) = f(g(x, yz))$
$= f(x + y; 2y, 5z)$
$= (e^{x+y}, \ln(x^2 + 5y^2 + 2xy + 1), 25z^2)$

Clearly $fog(x, y, z) \neq gof(x, y, z)$

Q. 5. Let $a = (1, 2, 3)$, $b = (-5, 3, -2)$, $c =$

(2, – 4, 1) be three points in R^3 find |2*b* – *a* + 3*c*|

Sol. We have

$$2b - a + 3c = (-10, 6, -4) - (1, 2, 3) + (6, -12, 3)$$
$$= (-5, -8, -4)$$
$$\therefore |2b - a + 3c| = \sqrt{(-5)^2 + (-8)^2 + (-4)^2}$$
$$= \sqrt{25 + 64 + 16}$$
$$= \sqrt{105}$$

Q. 6. Find the domain of the quotient of the functions *f* and *g* given by $f(x, y) = x^2 + 3y^2$ and $g(x, y) = 4x - 5y$.

Sol. The quotient of the function *f* and *g* defined by

$$(f/g)(x, y) = \frac{f(x,y)}{g(x,y)}$$
$$= \frac{x^2 + 3y^2}{4x - 5y}$$

and its domain is

$$R^2 \sim \{x \neq 0, y \neq 0, x \neq \frac{1}{4}, y \neq \frac{1}{5}\}$$

Q. 7. If *a* = (1, 2) and *b* = (2, 0) are two point in R^2 then find |*x* – *y*| and |3*x* – *y*| where *x* = *a* – 2*b* and *y* = 2*a* + *b*.

Sol. We have

$$x - y = a - 2b - 2a - b$$
$$= -a - 3b$$
$$= -(1, 2) - 3(2, 0)$$
$$= (-7, -2)$$
$$\therefore (x - y) = \sqrt{(-7)^2 + (-2)^2}$$
$$= \sqrt{49 + 4}$$
$$= \sqrt{53}$$

and
$$3x - y = 3(a - 2b) - 2a - b$$
$$= a - 7b$$
$$= (1, 2) - 7(2, 0)$$
$$= (-13, 2)$$
$$\therefore |3x - y| = \sqrt{(-13)^2 + (2)^2}$$
$$= \sqrt{169} + 4$$
$$= \sqrt{173}$$

Q. 8. Prove S1, S2, S3, S4 and S5 by using the corresponding properties of real numbers.

Sol. S_1 ; $x \in R^n$ where

$x = \{x_1, x_2, x_3 \ldots x_n\}$ then

$ax = (ax_1, ax_2 + ax_3 \ldots ax_n\}, ax \in R^n$

where $1 \leq i \leq r$

S_2 : $x = \{x_1, x_2 \ldots x_n\}$ and

$y = \{y_1, y_2 \ldots y_n\}$ then

$$a(x + y) = a(x_1, x_2, \ldots x_n) + (y_1, y_2 \ldots y_n)$$
$$= a(x_1 + y_1, x_2 + y_2, \ldots x_n + y_n)$$
$$= ax + ay.$$

S_3: $a(bx) = a(bx_1, bx_2, \ldots bx_n)$

$= a(bx)$

$= (ab)x.$ For $x \in R^n$, $ab \in R$.

S_4: $(a + b)x = (a + b)(x_1, x_2, x_3 \ldots x_n)$

$= (ax_1, ax_2, ax_3 \ldots ax_n) + b(x_1, x_2 \ldots x_n)$

$= ax + bx.$ For $ab \in R$, $x \in R^n$

S_5: $ax = 0 \ \forall \ x \in R^n$

when $x = \{1, 0, 0, \ldots 0\}$ then

$ax = 0$

$\Rightarrow a = 0$

This is true $\forall \ x \in R^n$

Q. 9. Let $e_i = (\delta_{i1}, \delta_{i2}, \ldots, \delta_{in})$, $1 \leq i \leq n$ where δ_{ij} is the Kronecker symbol, ($\delta_{ij} = 0$ if $i \neq j$, $\delta_{i,i} = 1$) be *n* vectors in R^n. Prove that any $x = (x_1, \ldots, x_n)$ in R^n can be written uniquely as

$$x = \sum_{i=1}^{n} x_i e_i$$

$e_1, \ldots, e_n$ are called the unit vectors along the co-ordinate axes.

Sol.
$$e_1 = (1, 0, \ldots)$$
$$e_2 = (0, 1, 0 \ldots)$$
$$e_3 = (0, 0, \ldots)$$

Let $x = (x_1, x_2, \ldots x_n) \in R^n$ then

$$x_1e_1 + x_2e_2 + \ldots x_ne_n = x_1(1, 0 \ldots) + x_2(0, 1, 0 \ldots) + \ldots + x_n(0, 0 \ldots)$$
$$= (x_1, x_2, \ldots x_n)$$

so $x \in R^n$, $x = \sum_{i=1}^{y} x_i e_i$

Q. 10. Let *e* = (1, 0), *f* = (1, 1) be in R^2.

Find | *x* – *y* |, | 2*x* – *y* |, | *x* | where *x* = *e* + *f*, *y* = 2*e* + 3*f*.

Sol.
$$x = e + f = (1, 0) + (1, 1)$$
$$= (2, 1)$$
$$y = 2e + 3f = 2(1, 0) + 3(1, 1)$$
$$= (2, 0) + (3, 3)$$
$$= (5, 3)$$
$$\therefore x - y = (2, 1) - (5, 3)$$
$$= (-3, -2)$$
$$\therefore |x - y| = \sqrt{(-3)^2 + (-2)^2}$$
$$= \sqrt{13}$$
$$|2x - y) = (|1, -1)|$$
$$= \sqrt{2}$$
$$|x| = |2, 1|$$
$$= \sqrt{5}$$